



**ST PAUL'S SCHOOL
JUNIOR SCHOLARSHIP EXAMINATION**

MAY 2011

MATHEMATICS

2 hours

Answer as many questions as you can in any order you wish.

Credit will be given for reasoning and working where appropriate.

**The total number of marks for this paper is 134.
The mark allocation is shown in brackets at the end of each part of each question.**

CALCULATORS MAY NOT BE USED.

1 If $a = -3$, $b = -8$ and $c = 2$, find the value of

(i) $(a + b)c + a$ [3]

(ii) $b - a^2 - c$ [3]

2 Calculate the values of the following, giving your answers as fractions in their lowest terms.

(i) $\frac{1\frac{1}{2} - \frac{2}{3}}{3\frac{1}{3} - 1\frac{1}{5}}$ (ii) $2\frac{5}{6} \times \frac{5}{17} \times \frac{6}{7} \times \frac{2}{5} \times 1\frac{3}{4}$ [7]

3 The whole number x satisfies both the inequalities

$$3x + 7 \leq 23 \quad \text{and} \quad 12 - 2x \leq 5$$

Find the possible values of x . [5]

4 Solve the equations:

(i) $\frac{x-1}{2} + \frac{2x}{5} = 4$ [3]

(ii) $\frac{3}{x+1} = \frac{8}{x}$ [3]

5 210 is an interesting number because when it is factorised into prime numbers:

$$210 = 2 \times 3 \times 5 \times 7$$

each prime numbers occurs on its own and they are consecutive.

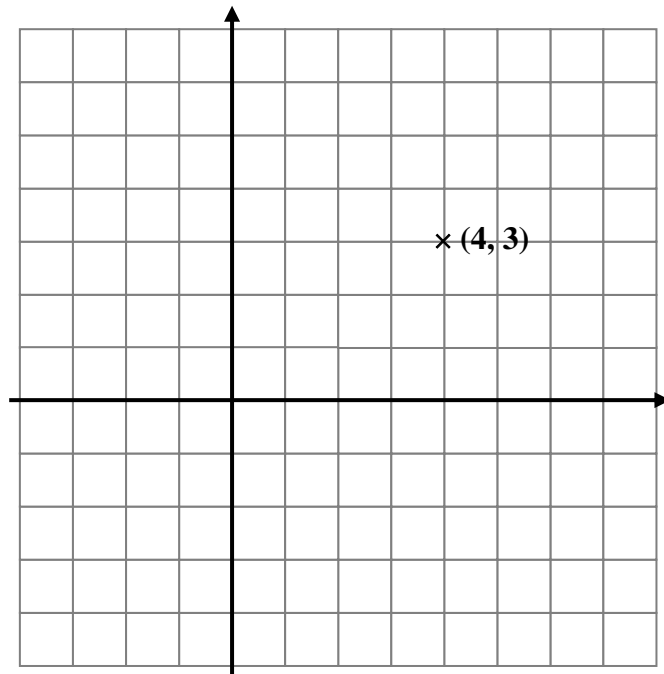
Factorise each of the following numbers into consecutive primes:

(i) 1155 (ii) 1001 [4]

- 6 (i) If a car has 27 litres of petrol in the tank it can travel 135 kilometres. How far can the car travel if it only has 22 litres of petrol? [3]
- (ii) If a car has p litres of petrol in the tank it can travel a kilometres. With v litres of petrol in the tank it can travel b kilometres. Find an expression for v in terms of p , b and a . [3]
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- 7 The mean of the numbers 3, 5, a , 2, 9 and 7 is 5. The mean of the numbers 3, 5, a , 2, 9, 7 and b is 6. Find the value of a and the value of b . [4]
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- 8 The point $(4, 3)$ is reflected in the line $y = x$ and the image is then rotated 90° clockwise about the point $(2, 1)$. Find the coordinates of the final image.



[3]

- 9 (a) Expand these expressions, simplifying your answer
- (i) $(x + 2)(2x - 1)$ [2]
- (ii) $(x + 2)(2x - 1)(x - 3)$ [3]
- (b) The expansion of $(3x + 2)(x - 4)(ax + 5)$ is $6x^3 - 5x^2 - 66x + b$. Find the value of a and the value of b . [3]
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TURN OVER

- 10** (i) Solve the simultaneous equations
- $$\begin{aligned}x &= y - 2 \\ 2x + 3y &= 16\end{aligned}$$
- [4]
- (ii) Bob went to the pie shop to buy some food for a party. He wanted to buy 12 pies in total and he only had £10 to spend. Beef pies were £1 each, vegetable pies were 70p each and chicken pies were 90p each. He spent all of his money and ended up buying 5 vegetable pies. How many beef pies did he buy? You must justify your answer. [5]
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- 11** (a) Find the next three terms in the following sequences:
- (i) 4, 8, 12, 16, 20, ... [1]
- (ii) 66, 59, 52, 45, ... [1]
- (iii) 1, 8, 14, 19, 23, ... [1]
- (iv) 4, 16, 21, 21, 18, 14, 11, ... [3]
- (b) Find the 100th term of the sequence 5, 8, 11, 14, ... [3]
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- 12** Simon is trying to work out the area of a circle from its radius. Unfortunately he uses the formula for the circumference of a circle by mistake. Despite his error he ends up with the correct answer anyway. Find the radius of the circle. [3]
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15 In this question $a * b$ is defined as $a * b = 2(a - b) + (a + b)$

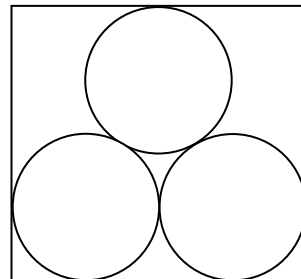
- (i) Find $6 * 7$ [2]
(ii) Find $-3 * \frac{3}{2}$ [2]
(iii) Solve $(2x) * x = 27$ [2]
(iv) If $m * n = 17$, find the value of $(m - 1) * (n - 1)$ [4]
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16 (i) £150 is reduced by 20% and the answer is reduced by 25%.

- (a) Find the final value. [2]
(b) What single percentage reduction overall is this? [3]
(ii) If $x\%$ of x is 4, find the value of x . [3]
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17 (i) An equilateral triangle has sides of length $2r$. Find the perpendicular height of the triangle, giving your answer in the form $r\sqrt{a}$. [3]

- (ii) Three circles of radius r are placed together so that their centres form an equilateral triangle. A rectangle is placed around them. Find an expression for the area of the rectangle, in terms of r .



[6]

18 A right-angled triangle T has sides 3 cm, 4 cm and 5 cm.

- (i) Find the area of the triangle. [1]

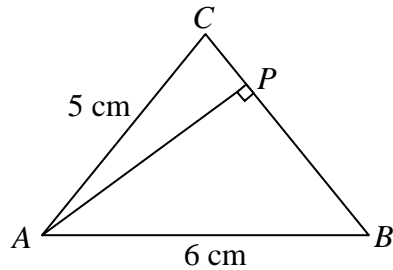
Another right-angled triangle U is an enlargement of triangle T with scale factor 1.2, so that the hypotenuse is of length 6 cm.

- (ii) Find the lengths of the other two sides. [2]

- (iii) Find the area of triangle U . [2]

Two copies of triangle T are joined together to make an isosceles triangle ABC of sides 5 cm, 5 cm, 6 cm.

P is the point on BC such that $\angle APB = 90^\circ$



- (iv) By considering the area of triangle ABC , show that $AP = 4.8$ cm. [3]

- (v) By using your previous answers, find the area of triangle APC . [4]

19 You are given the following two sums:

$$1 + 2 + 3 + \dots + 50 = 1275$$

$$\text{and } 1 + 2 + 3 + \dots + 100 = 5050$$

Using these results, find the following sums. *No credit will be gained for answers obtained by long methods.*

- (a) $51 + 52 + 53 + \dots + 100$ [3]

- (b) $2 + 4 + 6 + \dots + 100$ [3]

- (c) $1 + 3 + 5 + \dots + 99$ [3]

- (d) $1 - 2 + 3 - 4 + 5 - \dots + 99 - 100$ [3]

- (e) $100.01 - 100.02 + 100.03 - 100.04 + \dots - 101$ [3]

END OF PAPER

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