

General Certificate of Education  
January 2007  
Advanced Subsidiary Examination



**MATHEMATICS**  
**Unit Pure Core 2**

**MPC2**

Wednesday 10 January 2007 1.30 pm to 3.00 pm

**For this paper you must have:**

- an 8-page answer book
  - the **blue** AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Time allowed: 1 hour 30 minutes

**Instructions**

- Use blue or black ink or ball-point pen. Pencil should only be used for drawing.
- Write the information required on the front of your answer book. The *Examining Body* for this paper is AQA. The *Paper Reference* is MPC2.
- Answer **all** questions.
- Show all necessary working; otherwise marks for method may be lost.

**Information**

- The maximum mark for this paper is 75.
- The marks for questions are shown in brackets.

**Advice**

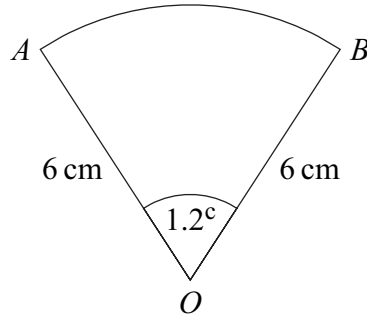
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.

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Answer **all** questions.

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- 1 The diagram shows a sector  $OAB$  of a circle with centre  $O$ .



The radius of the circle is 6 cm and the angle  $AOB$  is 1.2 radians.

- (a) Find the area of the sector  $OAB$ . (2 marks)
- (b) Find the perimeter of the sector  $OAB$ . (3 marks)

- 2 Use the trapezium rule with four ordinates (three strips) to find an approximate value for

$$\int_0^3 \sqrt{2^x} \, dx$$

giving your answer to three decimal places. (4 marks)

- 3 (a) Write down the values of  $p$ ,  $q$  and  $r$  given that:

(i)  $64 = 8^p$ ;

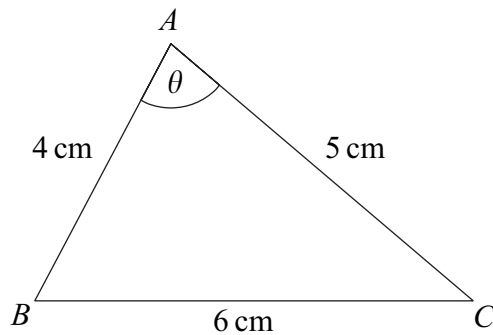
(ii)  $\frac{1}{64} = 8^q$ ;

(iii)  $\sqrt{8} = 8^r$ . (3 marks)

- (b) Find the value of  $x$  for which

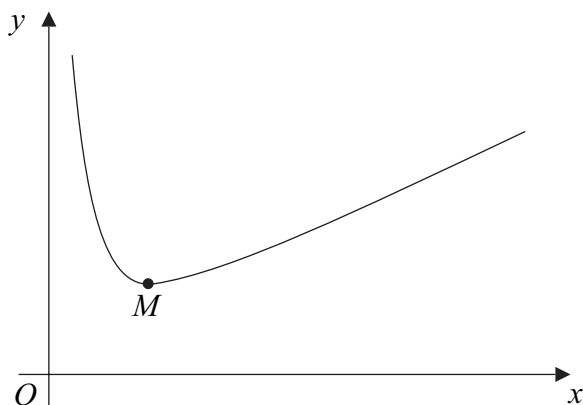
$$\frac{8^x}{\sqrt{8}} = \frac{1}{64} \quad (2 \text{ marks})$$

- 4 The triangle  $ABC$ , shown in the diagram, is such that  $BC = 6$  cm,  $AC = 5$  cm and  $AB = 4$  cm. The angle  $BAC$  is  $\theta$ .



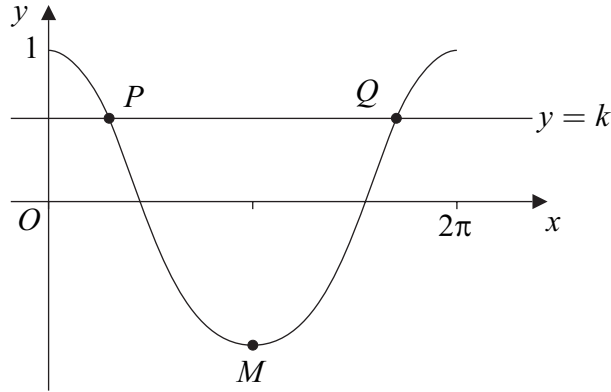
- (a) Use the cosine rule to show that  $\cos \theta = \frac{1}{8}$ . (3 marks)
- (b) Hence use a trigonometrical identity to show that  $\sin \theta = \frac{3\sqrt{7}}{8}$ . (3 marks)
- (c) Hence find the area of the triangle  $ABC$ . (2 marks)
- 5 The second term of a geometric series is 48 and the fourth term is 3.
- (a) Show that one possible value for the common ratio,  $r$ , of the series is  $-\frac{1}{4}$  and state the other value. (4 marks)
- (b) In the case when  $r = -\frac{1}{4}$ , find:
- (i) the first term; (1 mark)
- (ii) the sum to infinity of the series. (2 marks)

- 6 A curve  $C$  is defined for  $x > 0$  by the equation  $y = x + 1 + \frac{4}{x^2}$  and is sketched below.



- (a) (i) Given that  $y = x + 1 + \frac{4}{x^2}$ , find  $\frac{dy}{dx}$ . (3 marks)
- (ii) The curve  $C$  has a minimum point  $M$ . Find the coordinates of  $M$ . (4 marks)
- (iii) Find an equation of the normal to  $C$  at the point  $(1, 6)$ . (4 marks)
- (b) (i) Find  $\int \left( x + 1 + \frac{4}{x^2} \right) dx$ . (3 marks)
- (ii) Hence find the area of the region bounded by the curve  $C$ , the lines  $x = 1$  and  $x = 4$  and the  $x$ -axis. (2 marks)
- 7 (a) The first four terms of the binomial expansion of  $(1 + 2x)^8$  in ascending powers of  $x$  are  $1 + ax + bx^2 + cx^3$ . Find the values of the integers  $a$ ,  $b$  and  $c$ . (4 marks)
- (b) Hence find the coefficient of  $x^3$  in the expansion of  $\left(1 + \frac{1}{2}x\right)(1 + 2x)^8$ . (3 marks)

- 8 (a) Solve the equation  $\cos x = 0.3$  in the interval  $0 \leq x \leq 2\pi$ , giving your answers in radians to three significant figures. (3 marks)
- (b) The diagram shows the graph of  $y = \cos x$  for  $0 \leq x \leq 2\pi$  and the line  $y = k$ .



The line  $y = k$  intersects the curve  $y = \cos x$ ,  $0 \leq x \leq 2\pi$ , at the points  $P$  and  $Q$ .  
The point  $M$  is the minimum point of the curve.

- (i) Write down the coordinates of the point  $M$ . (2 marks)
- (ii) The  $x$ -coordinate of  $P$  is  $\alpha$ .  
Write down the  $x$ -coordinate of  $Q$  in terms of  $\pi$  and  $\alpha$ . (1 mark)
- (c) Describe the geometrical transformation that maps the graph of  $y = \cos x$  onto the graph of  $y = \cos 2x$ . (2 marks)
- (d) Solve the equation  $\cos 2x = \cos \frac{4\pi}{5}$  in the interval  $0 \leq x \leq 2\pi$ , giving the values of  $x$  in terms of  $\pi$ . (4 marks)

**Turn over for the next question**

**Turn over ►**

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9 (a) Solve the equation  $3 \log_a x = \log_a 8$ . *(2 marks)*

(b) Show that

$$3 \log_a 6 - \log_a 8 = \log_a 27 \quad (3 \text{ marks})$$

(c) (i) The point  $P(3, p)$  lies on the curve  $y = 3 \log_{10} x - \log_{10} 8$ .

$$\text{Show that } p = \log_{10} \left( \frac{27}{8} \right). \quad (2 \text{ marks})$$

(ii) The point  $Q(6, q)$  also lies on the curve  $y = 3 \log_{10} x - \log_{10} 8$ .

$$\text{Show that the gradient of the line } PQ \text{ is } \log_{10} 2. \quad (4 \text{ marks})$$

**END OF QUESTIONS**

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